On the Way towards Topology-Based Visualization of Unsteady Flow – The State of the Art

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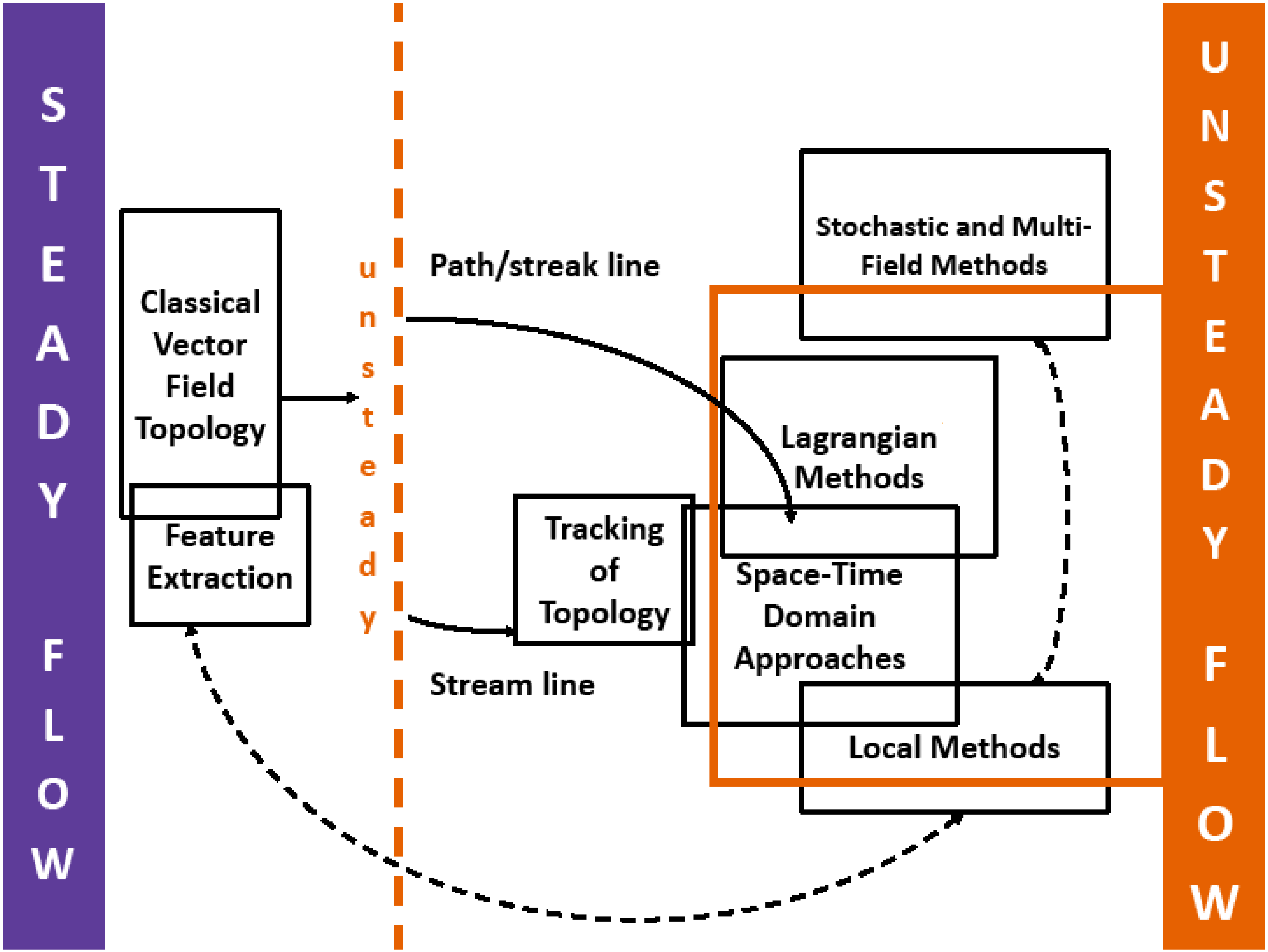
- **SemSeg - 4D Space-Time Topology for Semantic Flow Segmentation** is a research project founded the European Commission
- Collaboration between:
  - University of Bergen, Norway
  - VRVis research center Vienna, Austria
  - ETH Zürich, Switzerland
  - University of Magdeburg, Germany
- www.semseg.eu

Outline


STEADY FLOW
Classical Vector Field Topology
Feature Extraction
unsteady
Path/streak line
Stream line
Tracking of Topology
Stochastic and Multi-Field Methods
Lagrangian Methods
Space-Time Domain Approaches
Local Methods
UNSTEADY FLOW

Outline

- Introduction
- Classical vector field topology
- First steps towards time-dependent data
- Lagrangian methods
- Space-time domain approaches
- Local methods
- Statistical and Multi-Field Methods

On the Way towards Topology-Based Visualization of Unsteady Flow

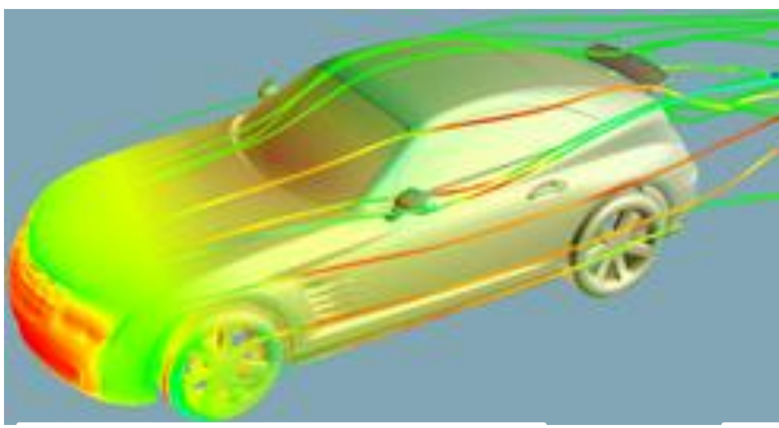
Introduction

Armin Pobitzer

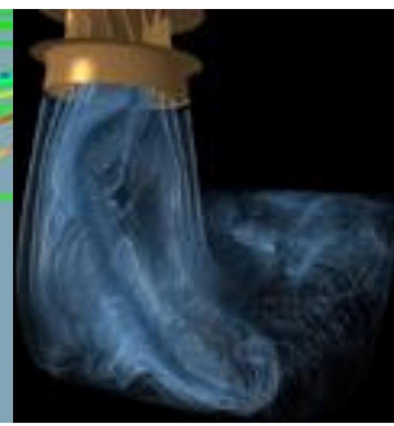
University of Bergen

What is "Flow"?

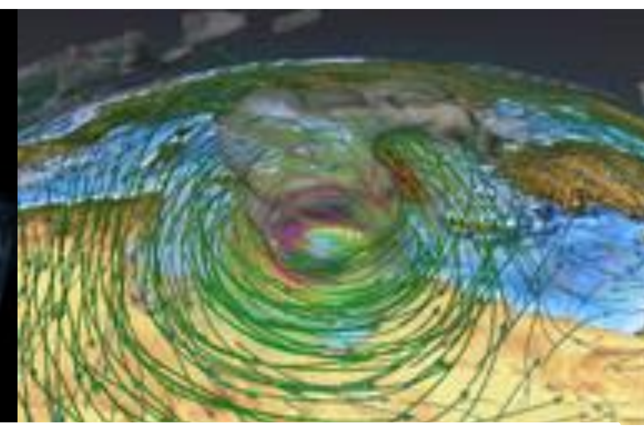
- Motion of liquids and gasses
- Mathematically modeled by PDEs (Navier-Stokes equations)
- For visualization: velocity field
  - generalization: any vector field



[avl.com]



[VATECH]



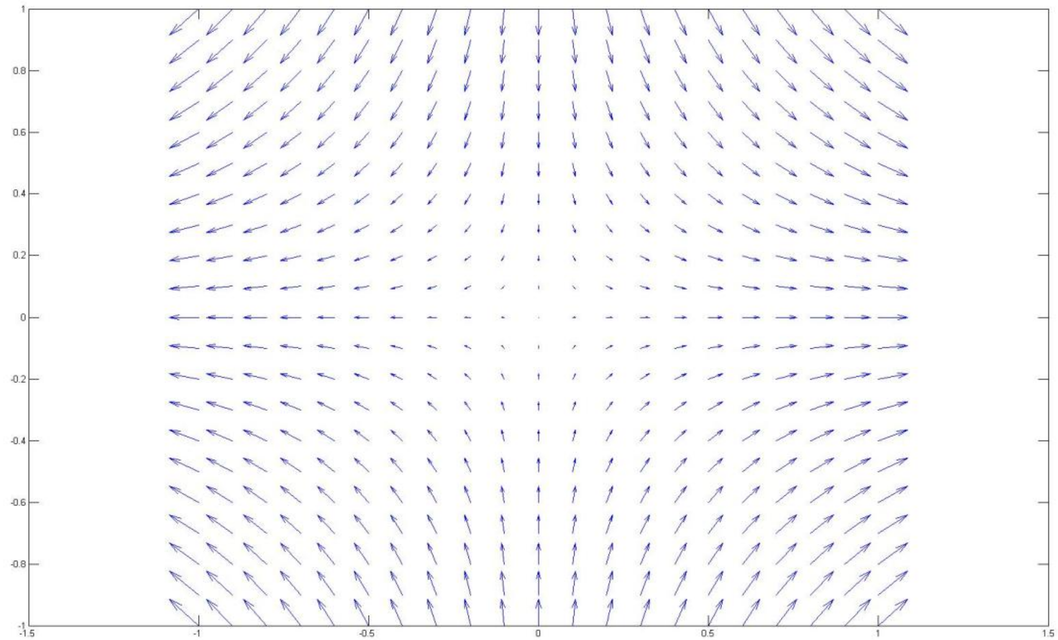
[M.Böttinger, DRMZ]

How does the Data look like?

- Vector field $\mathbf{v}: \mathbf{R}^n \rightarrow \mathbf{R}^n; \mathbf{x} \rightarrow \mathbf{v}(\mathbf{x})$
  - analytic (rare)
  - simulated $\rightarrow$ vectors on grid
- Dimenstions
  - n=2,3
- Time dependency
  - steady flow rare in nature!
  - time window
- What to visualize?
  - Example: analytic, n=2, steady
    $\mathbf{v}(\mathbf{x},\mathbf{y})=(\mathbf{x},-\mathbf{y})^T$

What to Visualize?

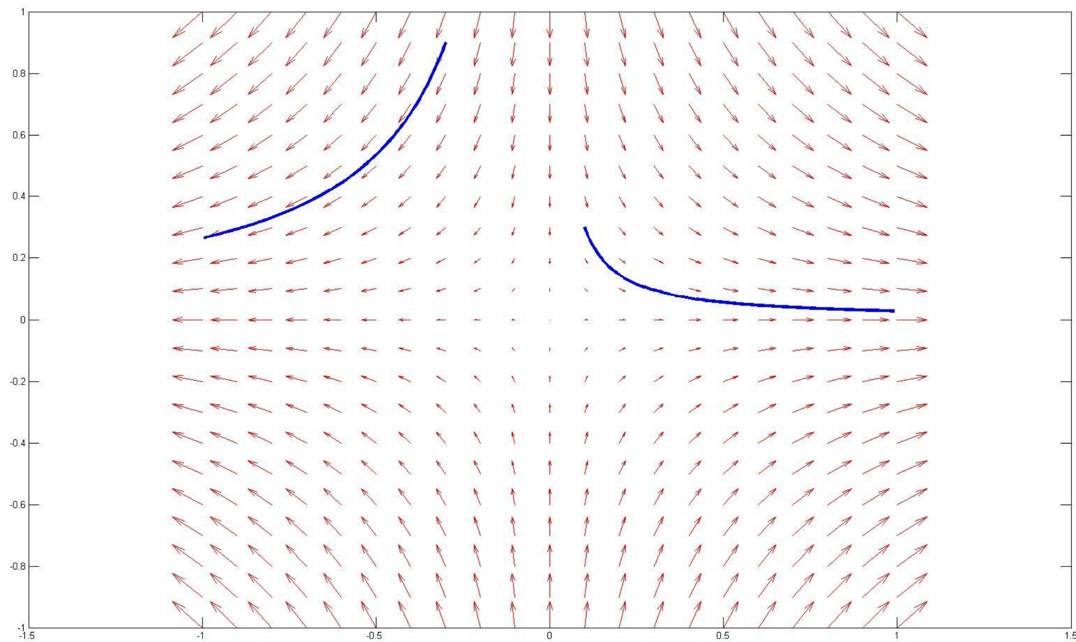
- Raw data
  - one possability:
    arrows
  - pro: - intuitive
    con: - little information on path of particles
    - clutter



$v(x,y)=(x,-y)^T$

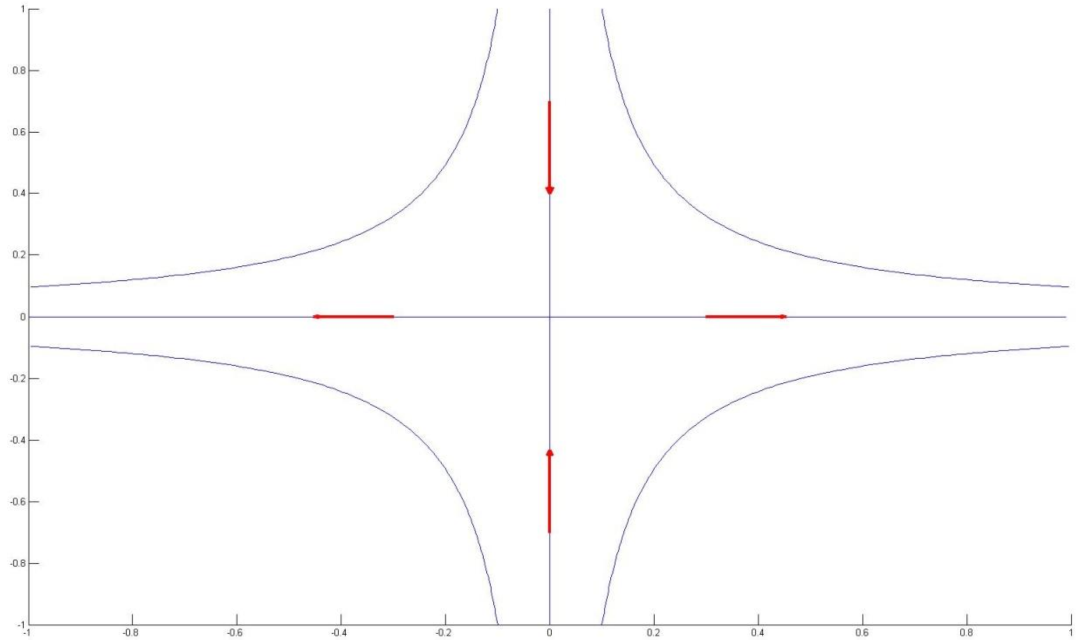
What to Visualize?

- Ingerational objects
  - one possability:
    path of particles
  - pro: - information on long term behavior
    con: - selective



What to Visualize?

- Topology: segmentation of flow in regions of different behavior (asymtocially)
  - pro: - solid mathematical theory
    - holistic
    - no clutter



Why bother?

www.thetruthaboutcars.com

On the Way towards Topology-Based Visualization of Unsteady Flow

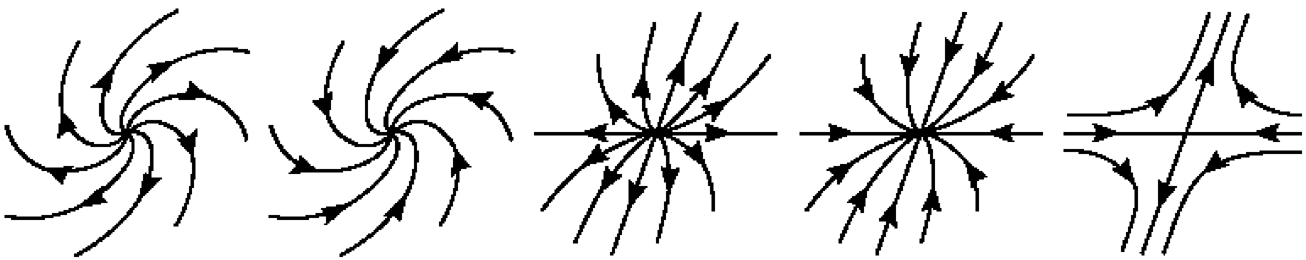
(Classical) Vector Field Topology

Vector Field Tolopolgy

- Based on theory of dynamical systems (H. Poincarè)
- Finding topological skeleton:
  - Computation of crtitical points
    i.e. find all $\mathbf{x}$ s.th. $\mathbf{v}(\mathbf{x}) = 0$
  - Classification of critical points
    based on eigenvalues of the gradient
  - Computation of the seperatrices
    i.e. integration from critical points in direction of the eigenvectors
  - Computation of higher order critical structures
    e.g. closed orbits
  - Classification of higher order critical structures

Finding the Topological Skeleton

- Computation of critical points
  - Analytical computation (piecewise linear fields)
  - Numerical computation
    - Newton–Raphson method
    - Subdivision methods
- Classification of critical points
  - Near critical point: $\mathbf{v}(\mathbf{x}+\mathbf{h})=\mathbf{v}(\mathbf{x})+J(\mathbf{x})\mathbf{h}+\ldots=J(\mathbf{x})\mathbf{h}+\ldots$

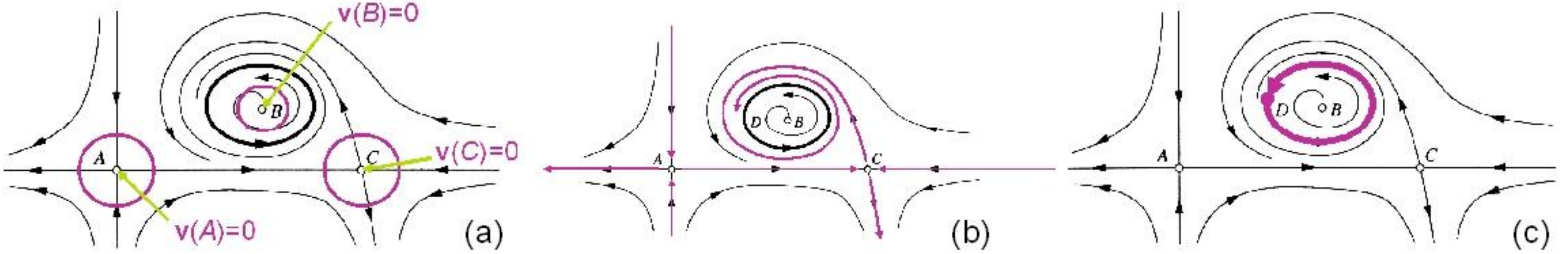


focus source	**focus sink**	**node source**	**node sink**	**saddle**	
$\mathrm{Im}(\lambda_{1,2})\neq 0$	$\mathrm{Im}(\lambda_{1,2})\neq 0$	$\mathrm{Im}(\lambda_{1,2})=0$	$\mathrm{Im}(\lambda_{1,2})=0$	$\mathrm{Im}(\lambda_{1,2})=0$	[GH83]
$\mathrm{Re}(\lambda_{1,2})>0$	$\mathrm{Re}(\lambda_{1,2})<0$	$\lambda_{1,2} > 0$	$\lambda_{1,2} < 0$	$\lambda_1\lambda_2 < 0$	

Finding the Topological Skeleton

- Computation of separatrices

  Integrate in direction **e** backward or forward in time according to the sign of the respective eigenvalue

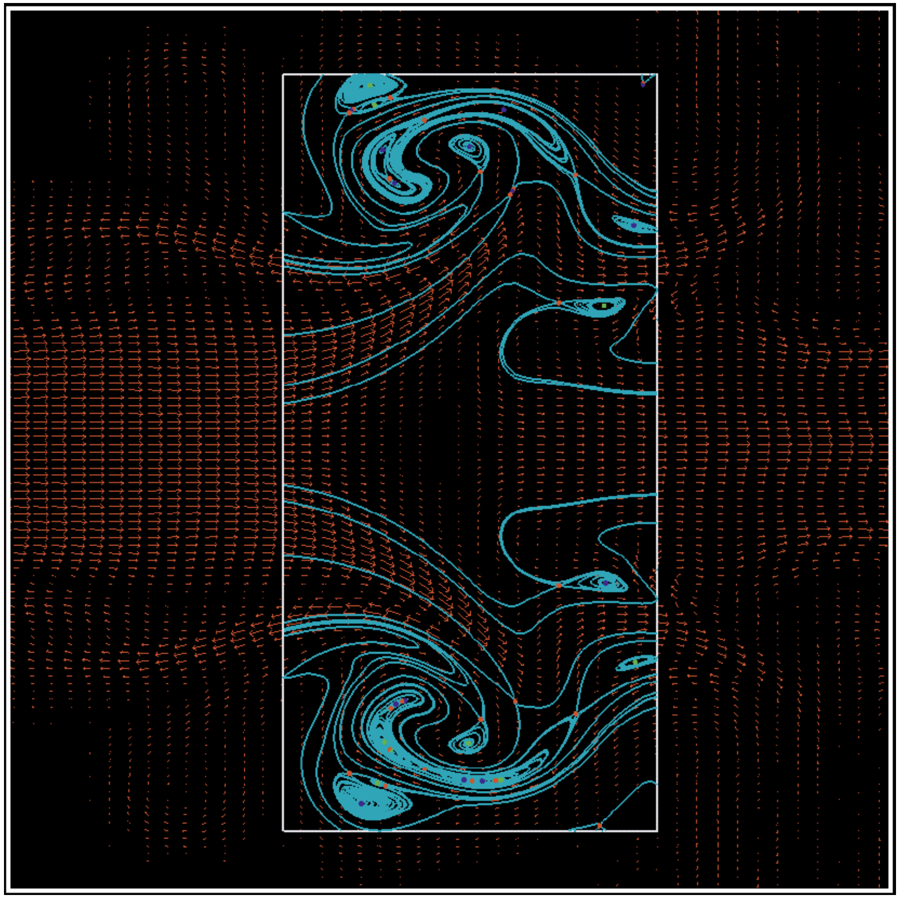


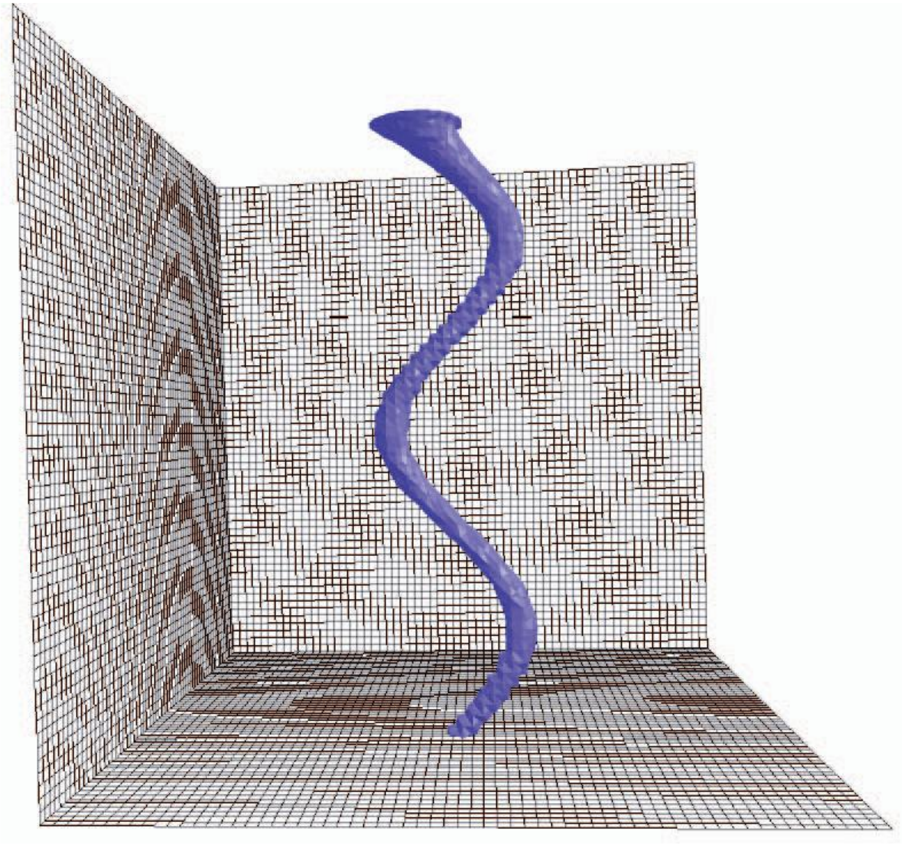
- Computation of higher order structures
- Classification of higher order structures
  repelling, attracting, saddle-like

[Asi93]

Separatices

[SHJK00]

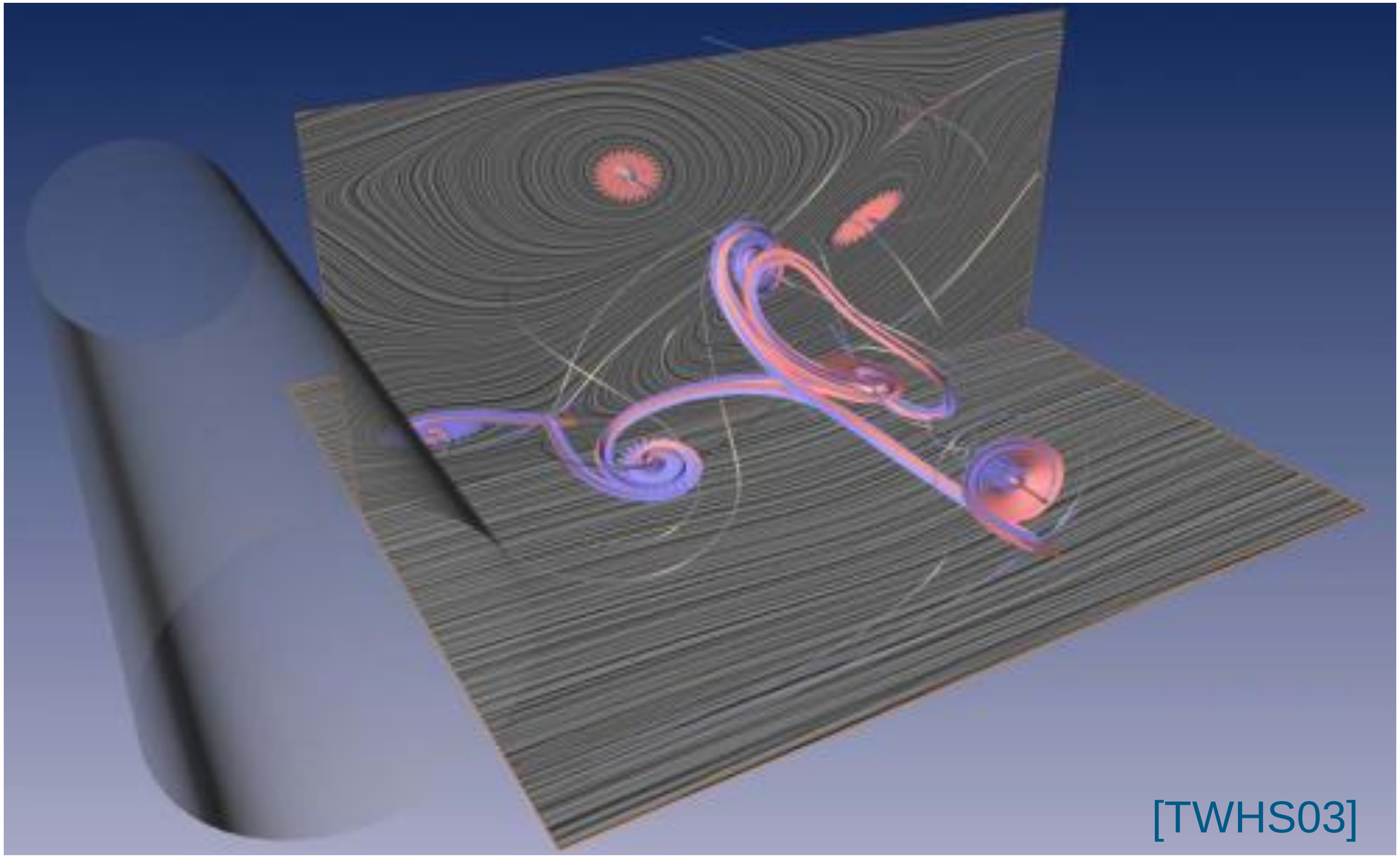




[MBS*04]

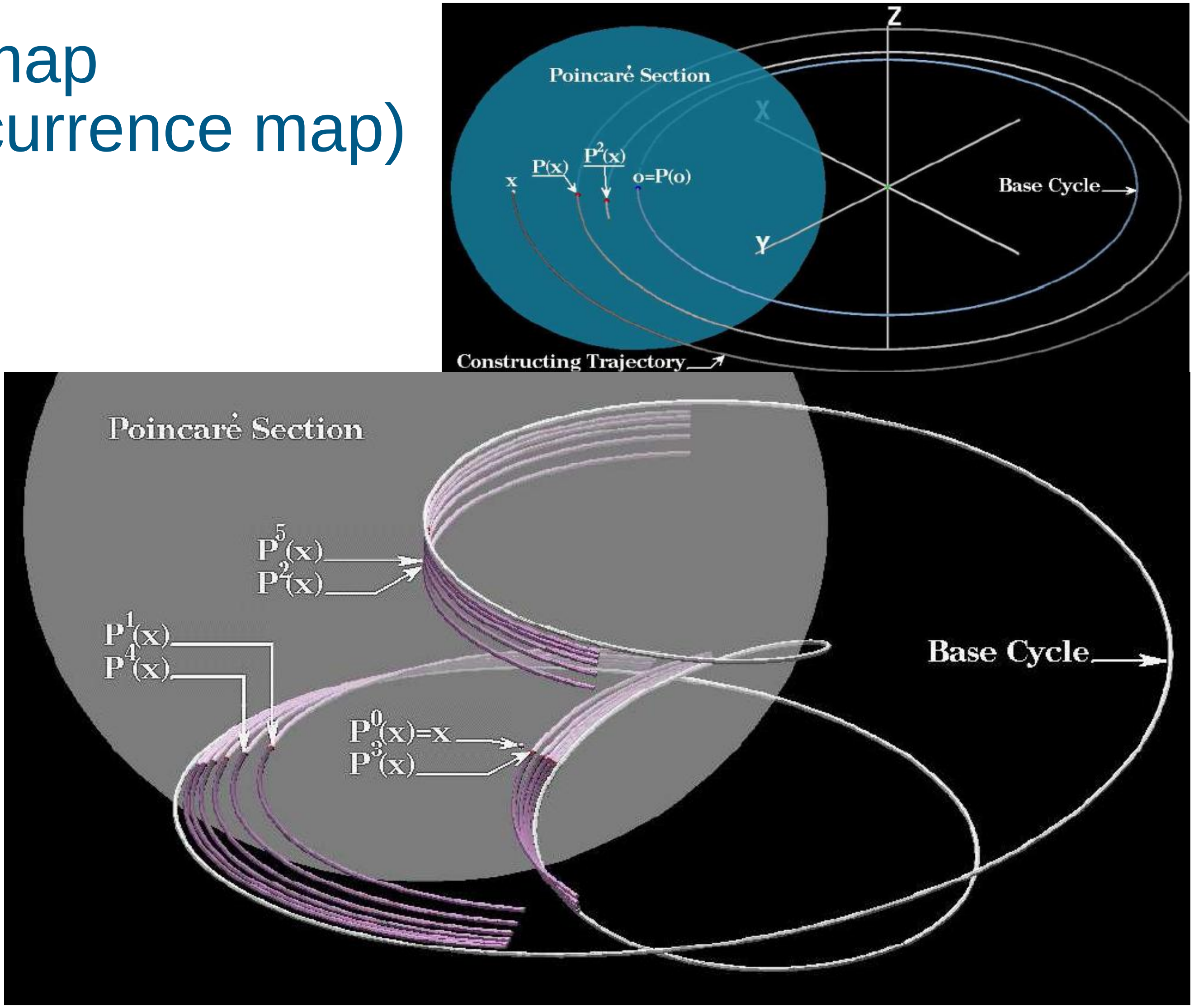
Separatrices

- 3D
  - some occlusion issues, but works



[TWHS03]

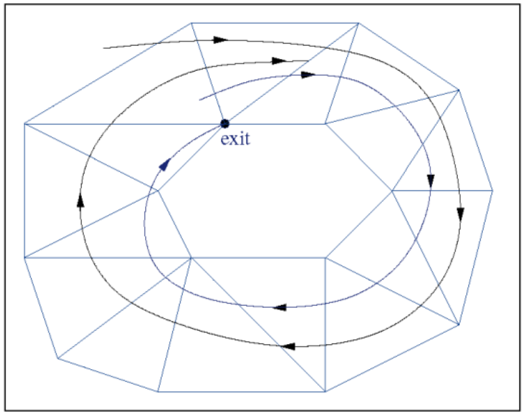
- Poincarè map
  (or first recurrence map)

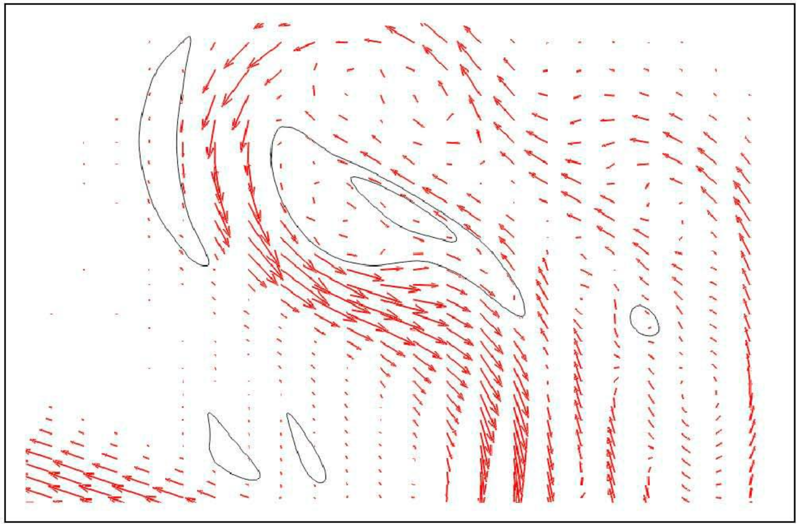


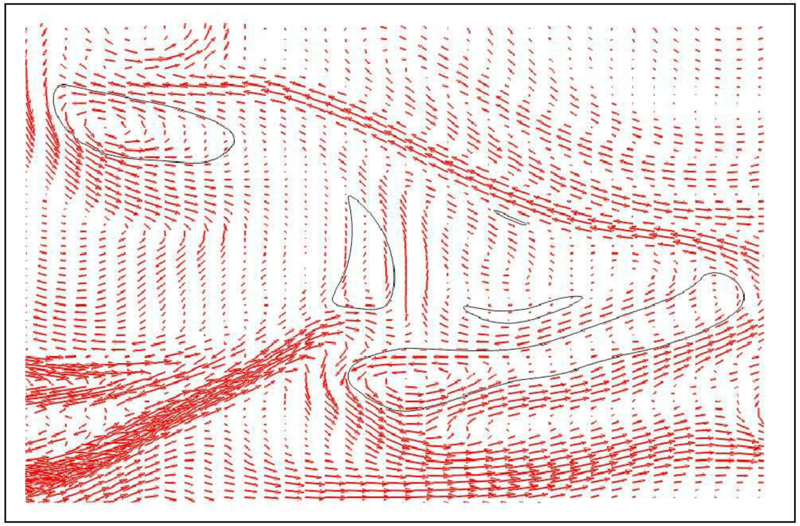
[LKG98]

Periodic Orbits

- Re-entering condition (based on theorem of Poincarè-Bendixon)



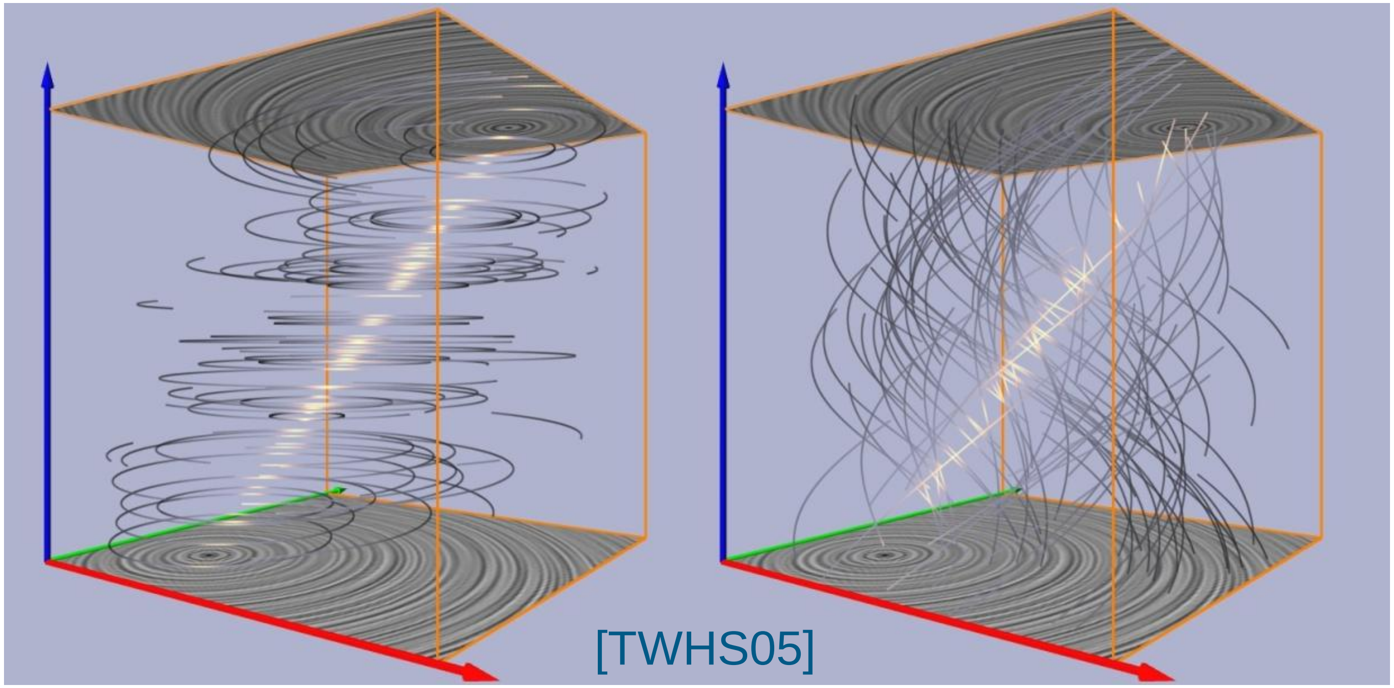




[WS01]

Time-dependent fields

- Different concepts
  - streamline:
    time-dependent flow = time-stack of steady
  - pathline:
    path of (massless) particle

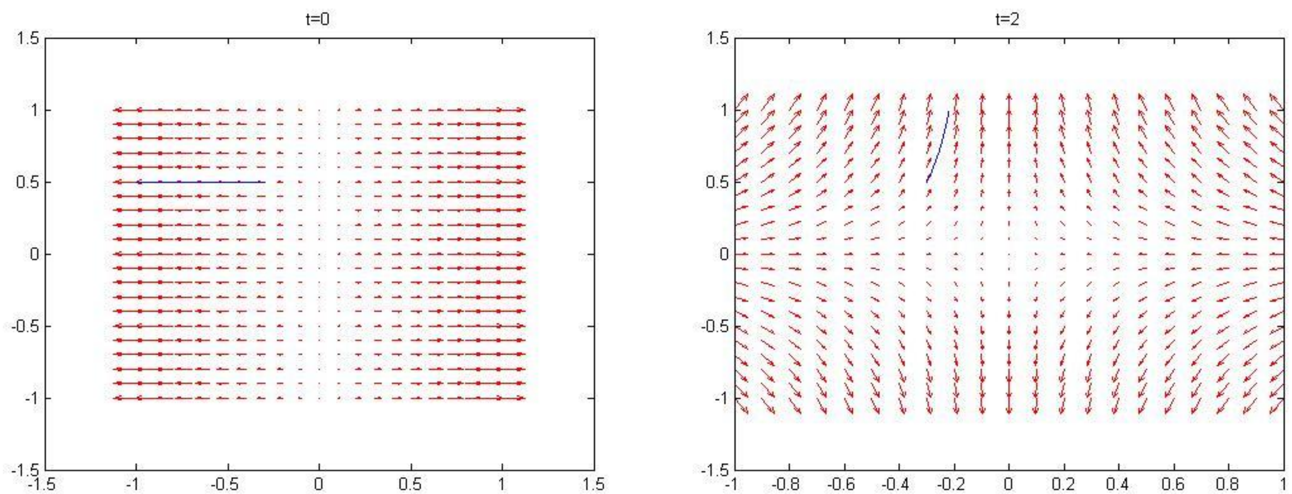


[TWHS05]

Streamline vs. Pathline

- Streamline
  - solution of initial value problem

    $\mathbf{x}'(t)=\mathbf{v}(\mathbf{x}(t),s), \qquad \mathbf{x}(0)=\mathbf{x}0$
  - topological segmentation of each time step s
  - physical interpretation questionable

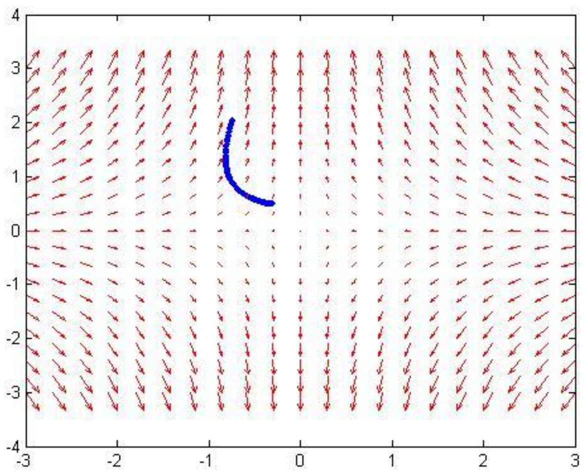


$v(x,y,t)=(x*\cos(t),y*\sin(t))^T$

Streamline vs. Pathline

- Pathline
  - solution of initial value problem

    $\mathbf{x}'(t)=\mathbf{v}(\mathbf{x}(t),t), \qquad \mathbf{x}(0)=\mathbf{x}0$
  - spacial intersection
  - no theory for segmentation



$v(x,y,t)=(x*\cos(t),y*\sin(t))^T$

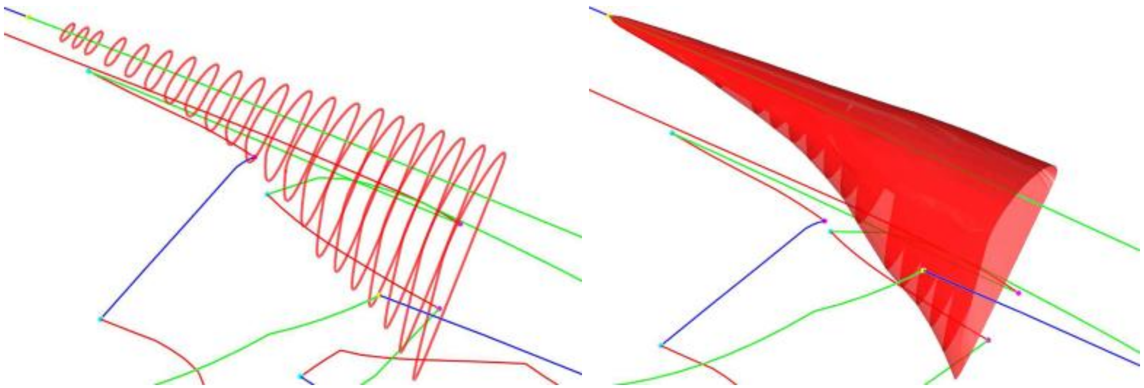
Pathline seeded at $(-0.3, 0.5)^T$ at time t=0.
Integration time [0,2].
Vector field at t=2 in background

First steps towards time-dependent data

Tracking of Topology

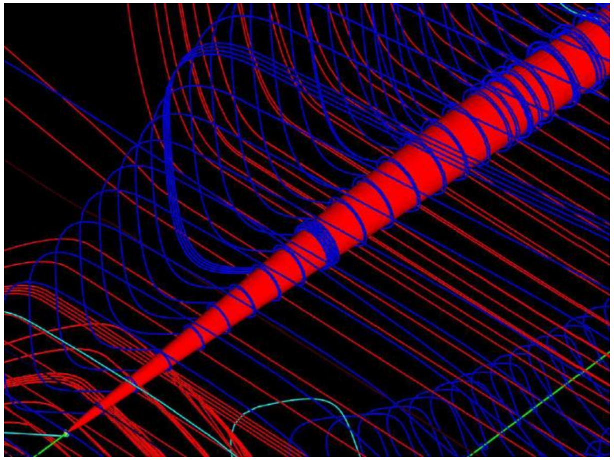
- Extract vector field topology for every time-slice
- Indentify corresponding stuctures in adjacent time steps

[WSH01]

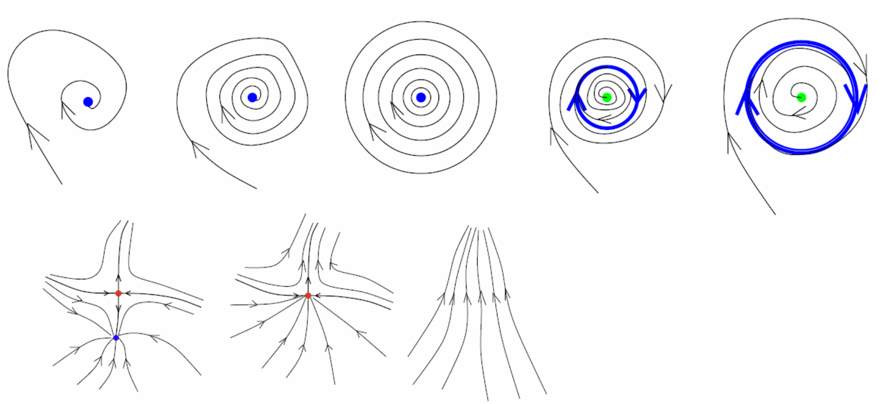


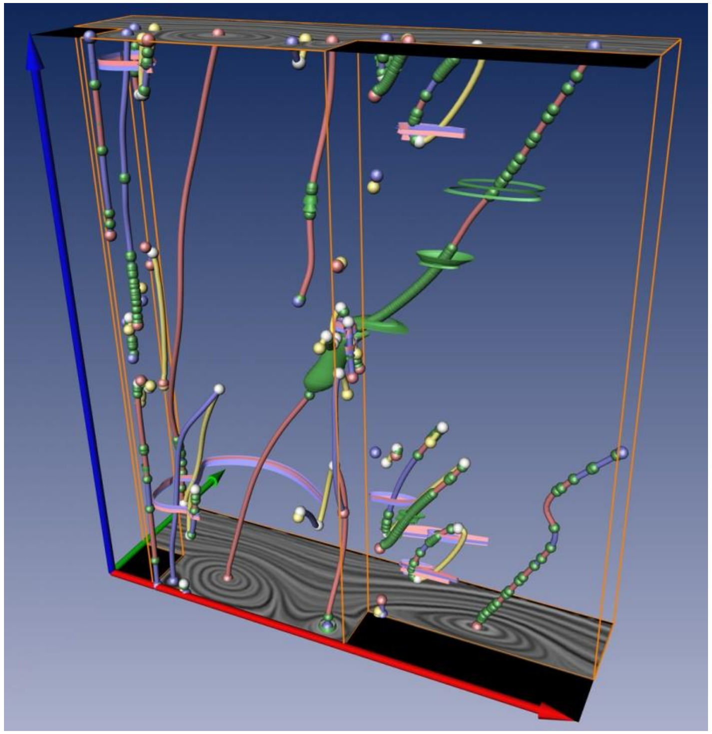
- Extracted geometry does not segment flow!

Bifurcations



[TSH01b]

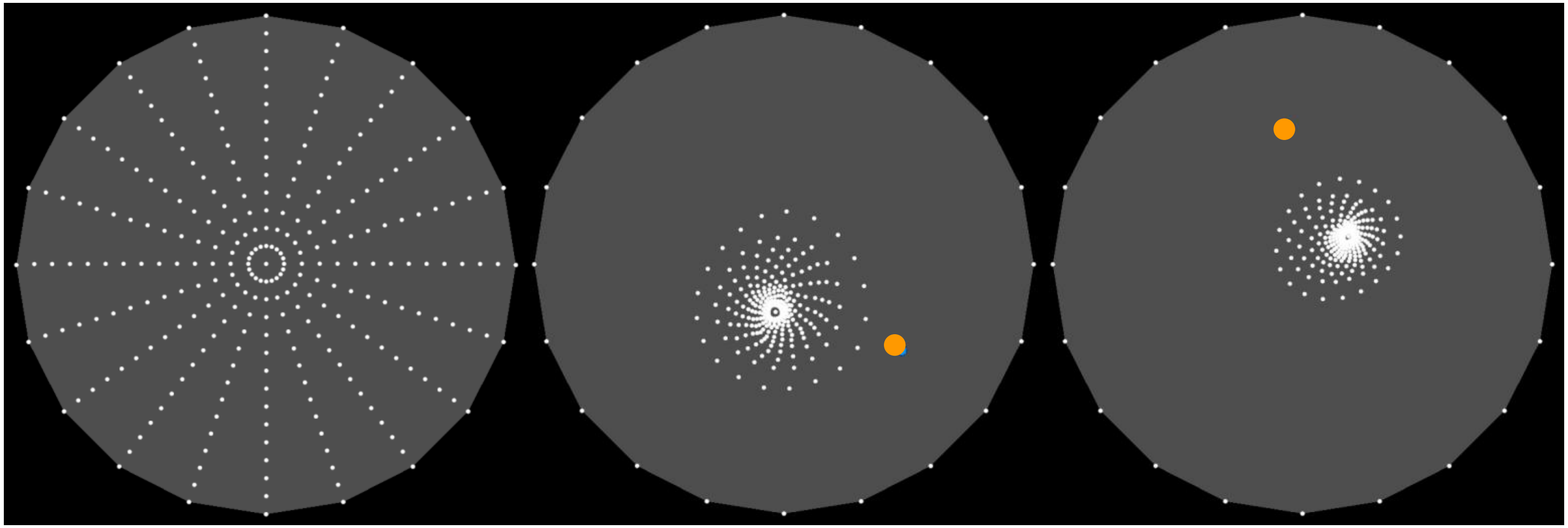




[TWHS05]

Deficiency of VFT for unsteady flow

- Only theoretically justified if the field is “almost” steady [Perry and Chong ‘94]
- Extracted structures may not have the claimed properties



[WCW*09]

On the Way Towards Topology-Based Visualization of Unsteady Flow

Lagrangian Methods

Benjamin Schindler

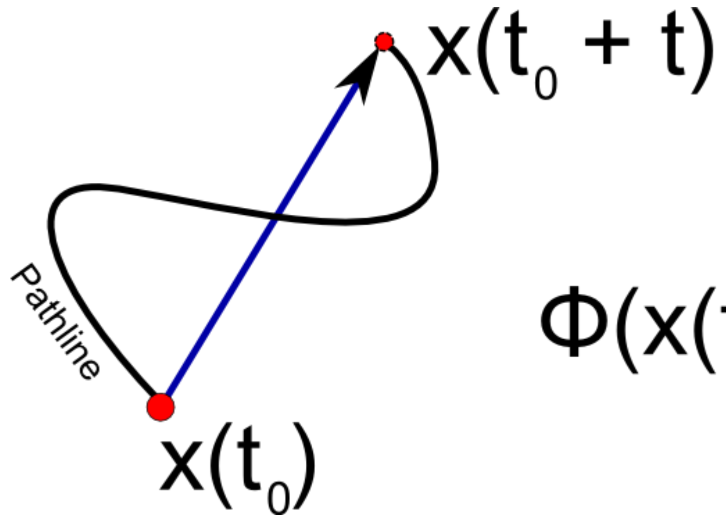
ETH Zürich

Contents

- Finite Time Lyapunov Exponent (FTLE) based methods
  - Introduction
  - FTLE as Lagrangian Coherent Structure (LCS)
  - Ridge computation
  - Evaluation
- Different Lagrangian feature detectors

Finite Time Lyapunov Exponent (FTLE)

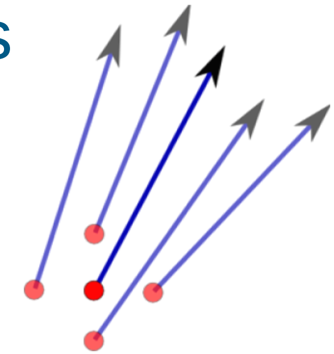
- Measure for flow separation (or contraction) over time
- Made popular by the works of Haller [Hal01, Hal02]
- Based on the flow map:



$$\Phi(x(t_0);\, t_0,\, t) = x(t_0 + t)$$

Finite Time Lyapunov Exponent (FTLE)

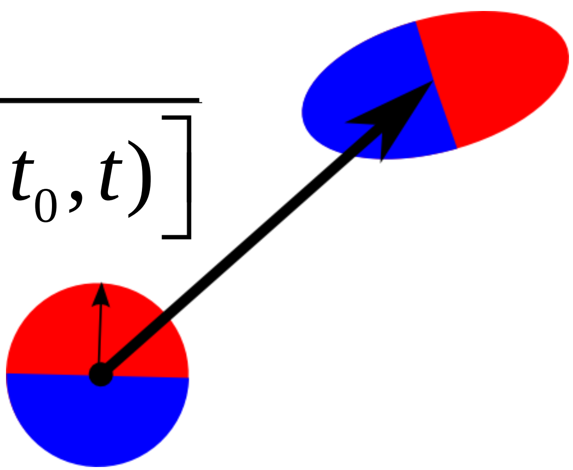
- Repelling is measured using the flow map gradient
  - Usually calculated using finite differences



$$\nabla\Phi(x(t_0);t_0,t)$$

- Maximal repelling occurs in the direction of the maximal eigenvalue of the squared flow map gradient

$$\varepsilon_{t_0}^{t}(x) = \sqrt{\lambda_{\max}\left[\nabla\Phi(x;t_0,t)^T\nabla\Phi(x;t_0,t)\right]}$$



Finite Time Lyapunov Exponent (FTLE)

- Recall Formula for maximal repelling

$$\varepsilon_{t_0}^{t}(x) = \sqrt{\lambda_{\max}\left[\nabla\Phi(x;t_0,t)^T\,\nabla\Phi(x;t_0,t)\right]}$$

- FTLE is defined as

$$\Lambda(t,t_0,x) = \log\left[\lambda_{\max}\;\;\nabla\Phi(x;t_0,t)^T\,\nabla\Phi(x;t_0,t)\;\;\right]^{\frac{1}{2}(t-t_0)}$$

- The local maxima of Λ coincide with the field ε

Finite Time Lyapunov Exponent (FTLE)

- Haller then defines Lagrangian Coherent Structures (LCS) as the height ridges of the field Λ
- Height Ridge: Maximum in at least one direction

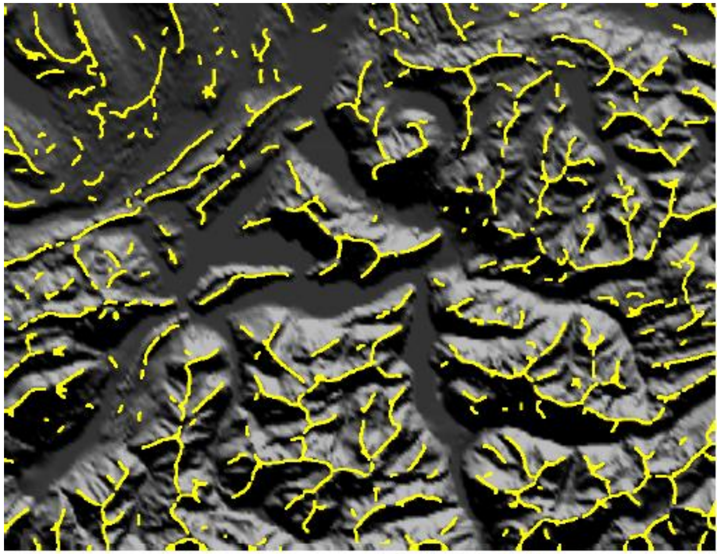
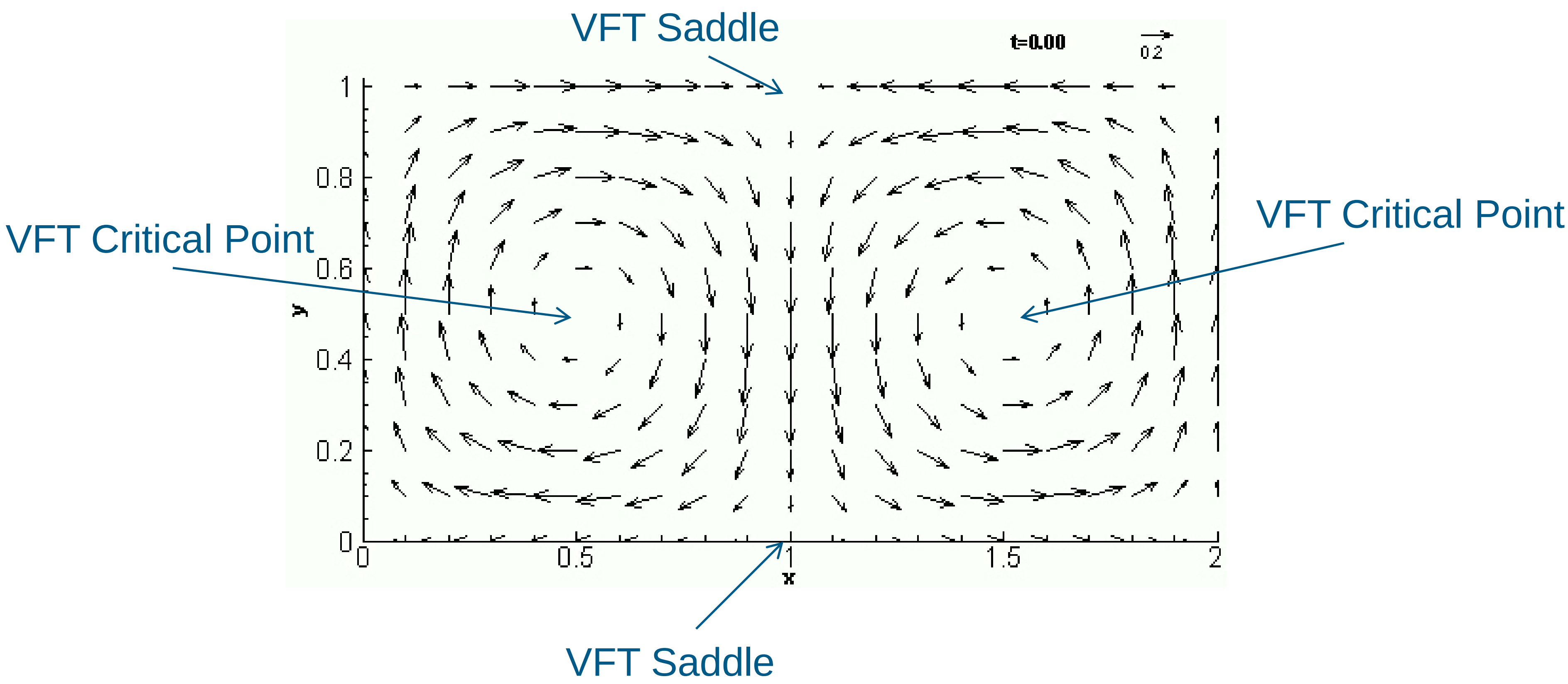


Image credit: P. Majer

- Attracting LCS obtained by calculating FTLE backwards in time

Finite Time Lyapunov Exponent (FTLE)

- Shadden et al. [SLM05] applied FTLE to the „double gyre“ example (among others)



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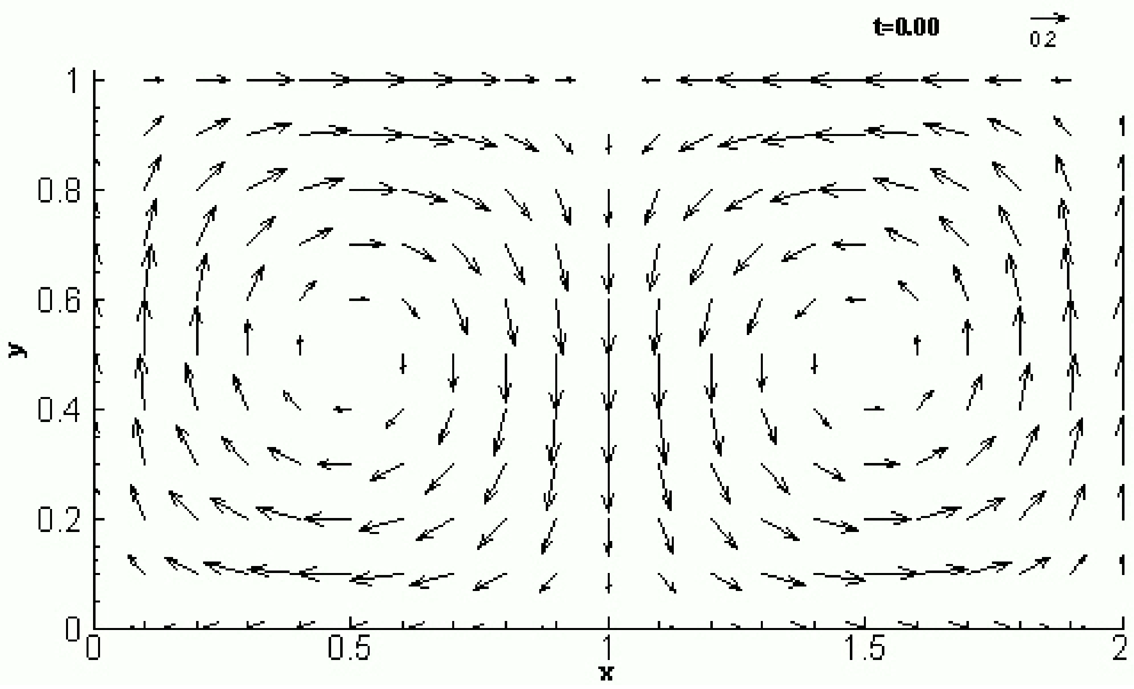

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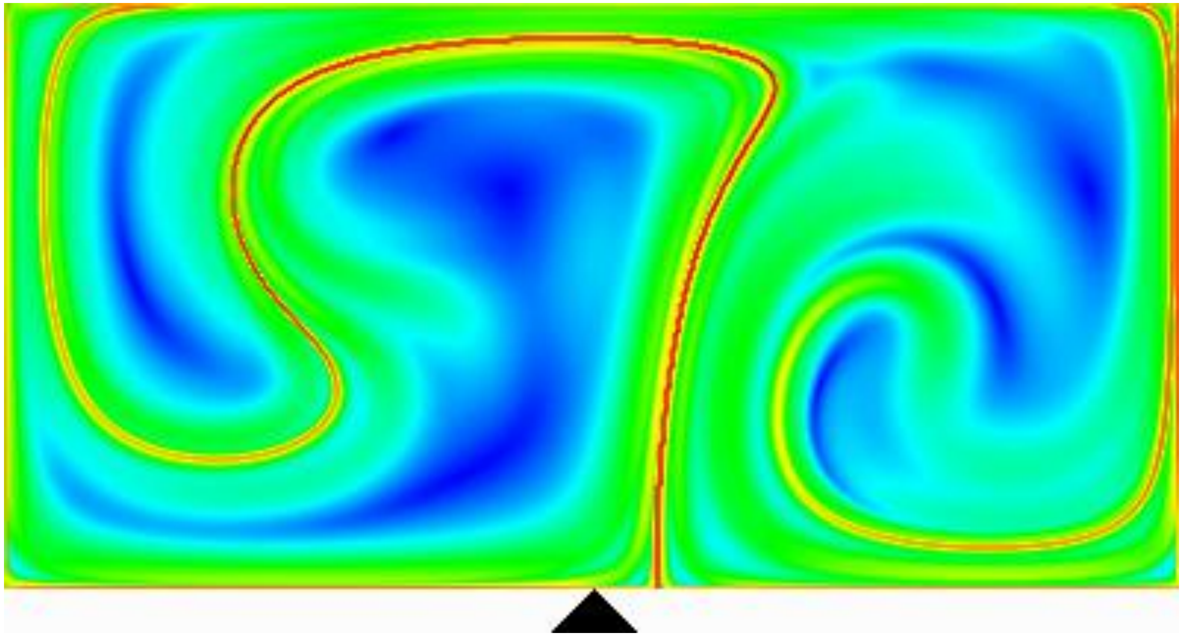


Finite Time Lyapunov Exponent (FTLE)

- Shadden et al. [SLM05] applied FTLE to the „double gyre“ example (among others)
  - Showed that particles seeded on the ridge follow it
  - Analytic formula for flux through the FTLE ridge

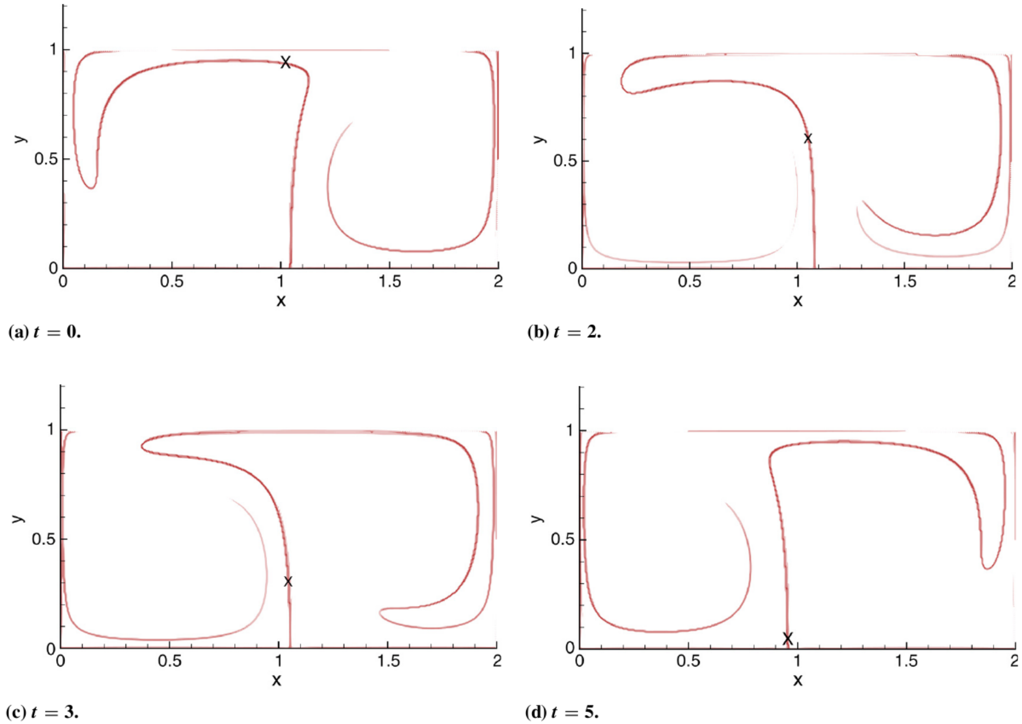


Image: Shadden 2005

FTLE visualization

- Garth et al. [GLT*09] Direct FTLE visualization using 2D Transferfunction
- [GGTH07] 3D FTLE computed as 2D in the plane orthogonal to the velocity
- Ridge computation is avoided by volume rendering

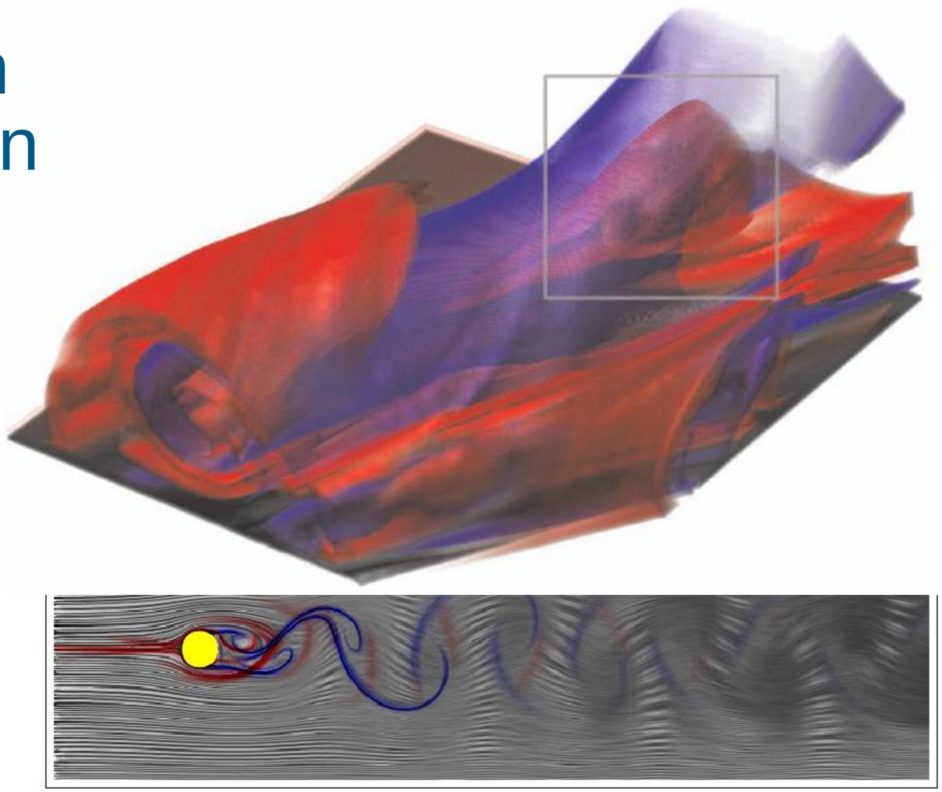


Image: Garth 2007

FTLE Ridge extraction

- Sadlo et al. [SP07a] FTLE height ridge calculation
  - Based on adaptive mesh refinement
  - Starts on a coarse grid and refines cells containing the ridge
  - Ridge extraction based on Hessian
  - Filtering of features required

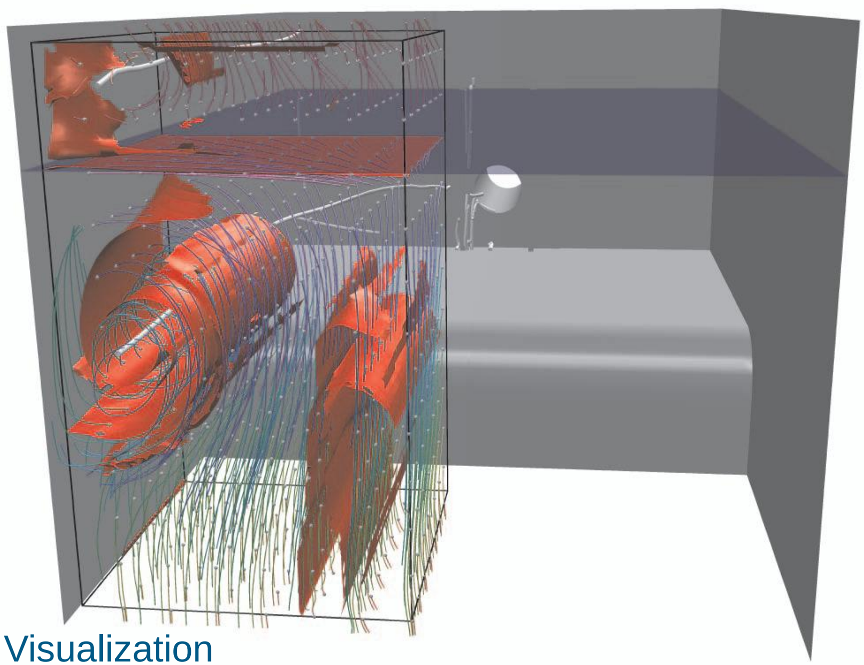
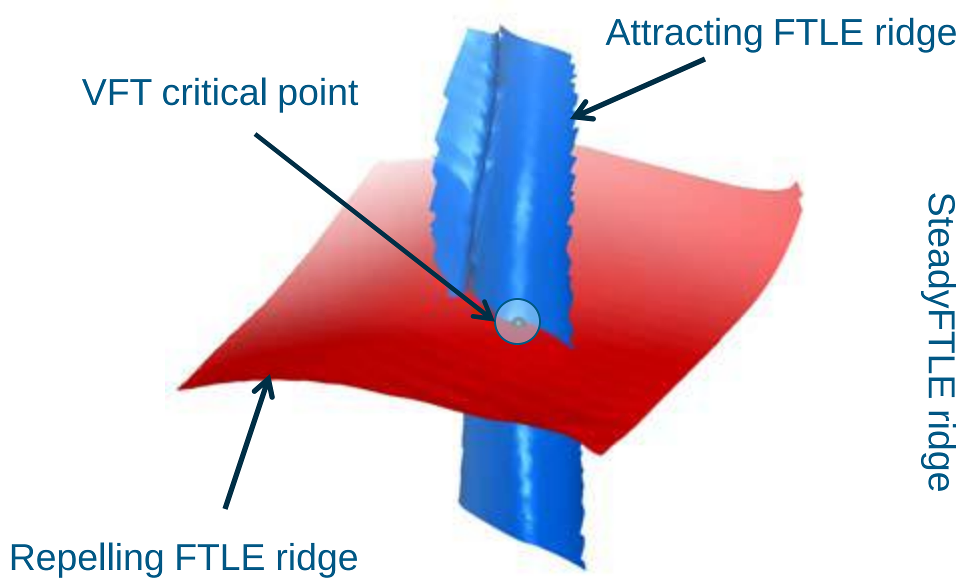
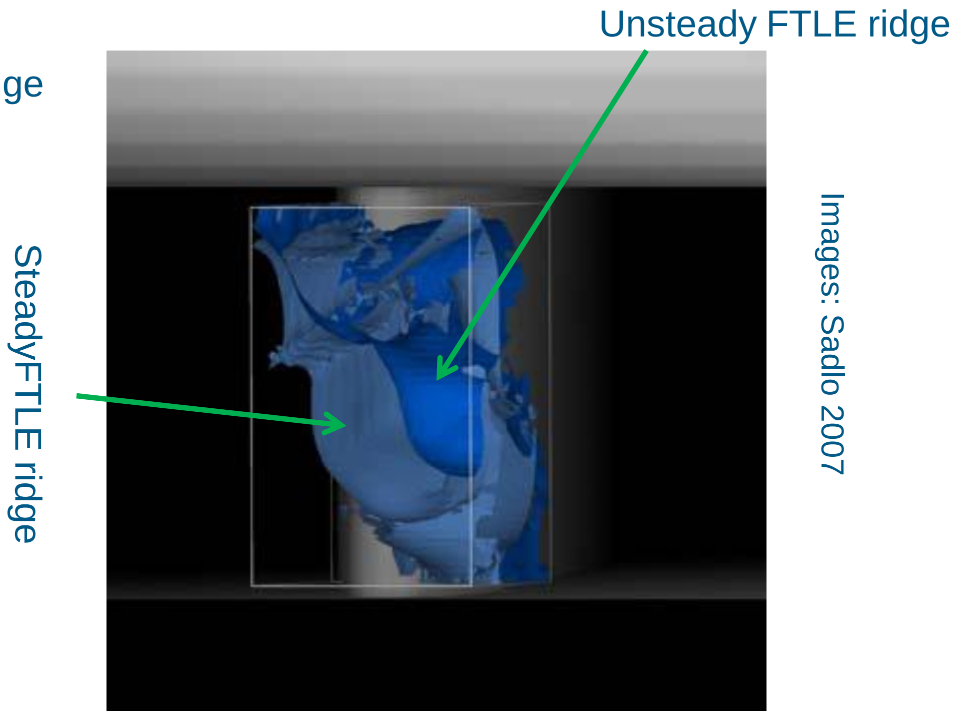


Image: Sadlo 2007

FTLE Evaluations

- Sadlo et al. [SP09] compares VFT to steady FTLE (FTLE computed on streamlines) and to unsteady FTLE
  - Steady FTLE very similar to VFT
  - Unseady FTLE works better than steady FTLE





Images: Sadlo 2007

FTLE Limitations

- Recall FTLE definition

$$\Lambda(t, t_0, x) = \log\left[\lambda_{\max} \quad \nabla\Phi(x; t_0, t)^T \nabla\Phi(x; t_0, t) \quad \right]^{\frac{1}{2}(t-t_0)}$$

- Cauchy-Green tensor in the square-root
- Rotational information is discarded when using FTLE
  - As a result, FTLE has limitations for vortex detection
- FTLE only gives information about flow separation – gives only limited information w.r.t. to VFT
- Effect of the choice of time window has not been studied sufficiently

Other Lagrangian Feature Detectors

- Fuchs et al. [FPS08] local vortex detectors for steady flow can be adapted by applying Lagrangian smoothing
- An objective definition of a vortex [Hal05]
  - Measure the time a trajectory spends in M_z
  - M_z is a cone in strain acceleration basis
  - Objective – i.e. Galilean in nder rotating frames of referenc

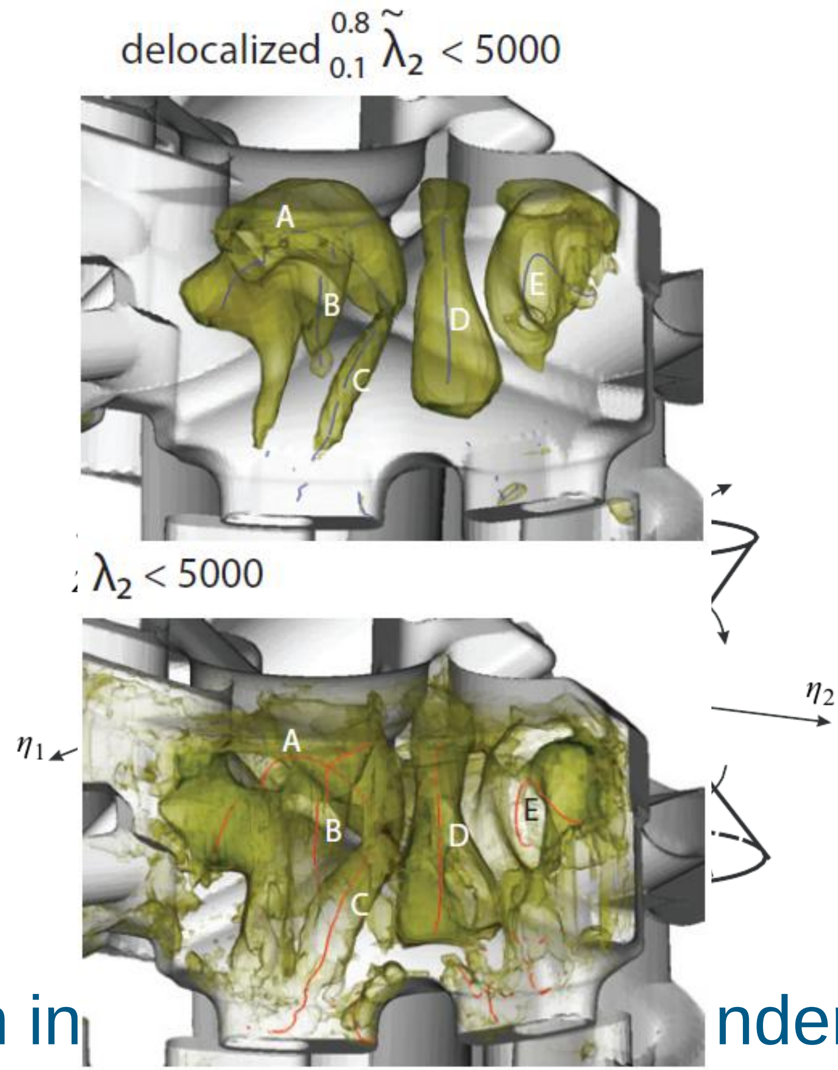


Image: Fuchs 2008

Image: Haller 2005

Other Lagrangian Feature Detectors

- Kasten et al. 2009 [KHNH09]
  - Unsteady critical points: Minima of the acceleration
  - Galilean invariant
  - Filtering based on long-livingness of critical points

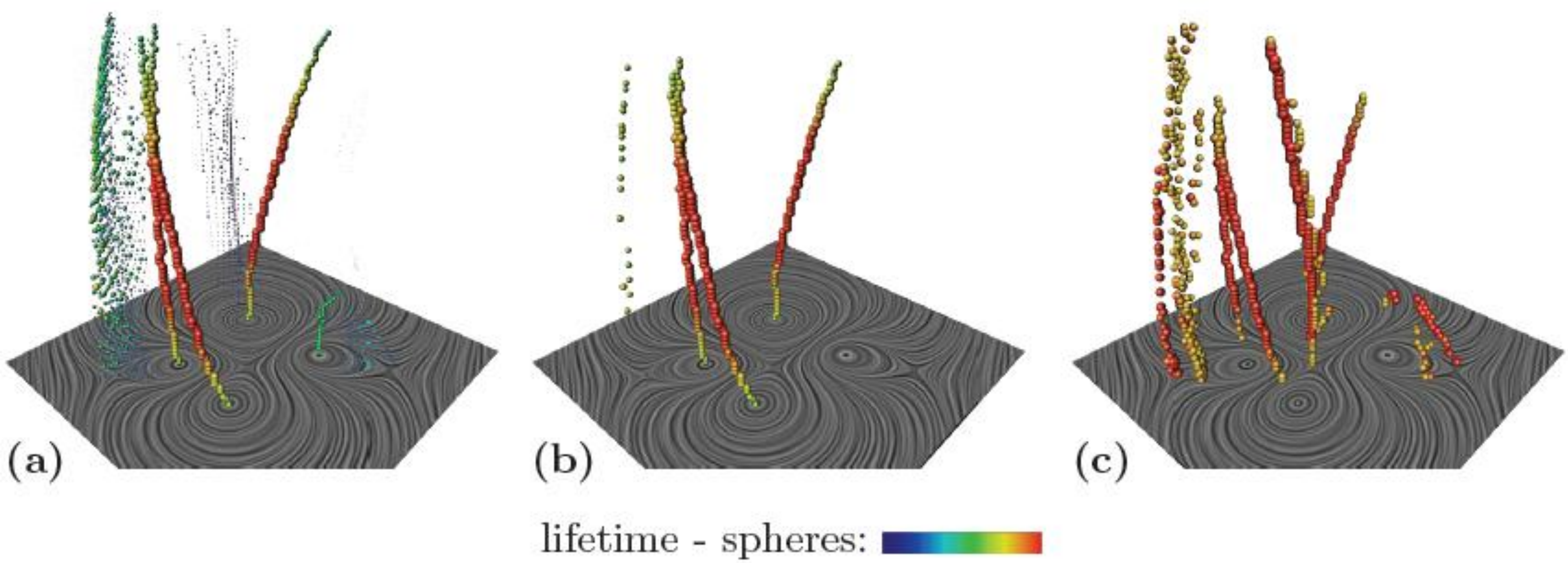


Image: Kasten 2009

On the Way Towards Topology-Based Visualization of Unsteady Flow

Space-time Domain Approaches

Alexander Kuhn

University of Magdeburg

Space-time Domain Approaches

- Approach to handle time-dependent data:
  - lift problem to higher dimension
    - time as additional space dimension
    - unsteady case → steady case
  - consider path- and streamlines
  - space and time can be handled in one set
  - extendable to arbitrary dimensions

Space-time Domain Approaches

- Formal definition:
  - Given time-dependent 2D vector field

$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$

  - Streamlines:

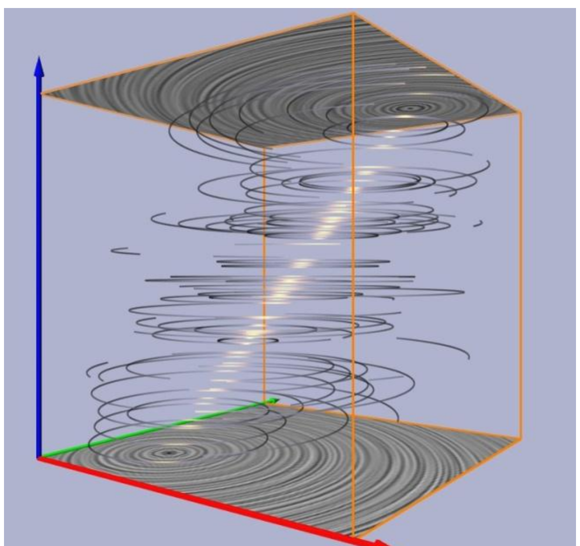
$$\mathbf{s}(\mathbf{x},t) = \begin{pmatrix} v(\mathbf{x},t) \\ u(\mathbf{x},t) \\ 0 \end{pmatrix}$$

  - Pathlines:

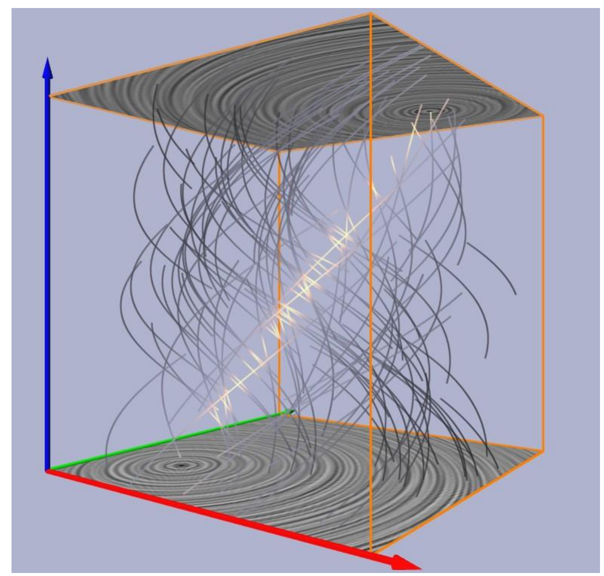
$$\mathbf{p}(\mathbf{x},t) = \begin{pmatrix} u(\mathbf{x},t) \\ v(\mathbf{x},t) \\ 1 \end{pmatrix}$$

Space-time Domain Approaches

- Example vectorfield [TWHS05]
  - Streamline:
    - no physical interpretation
  - Pathline:
    - path of (massless) particle



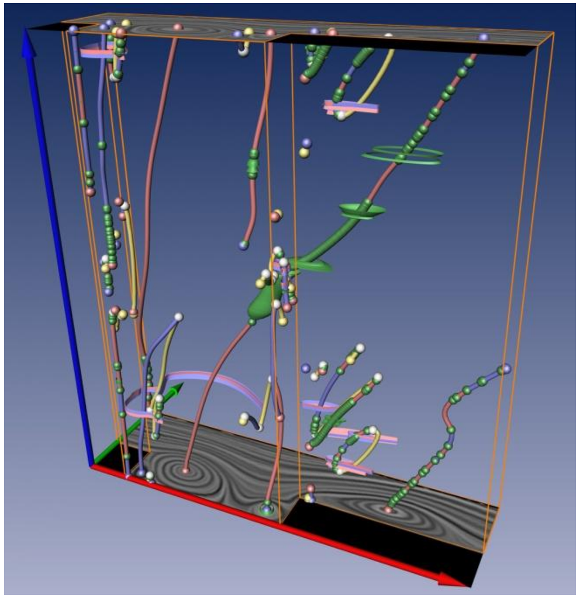
$\mathbf{s}(\mathbf{x}, t)$

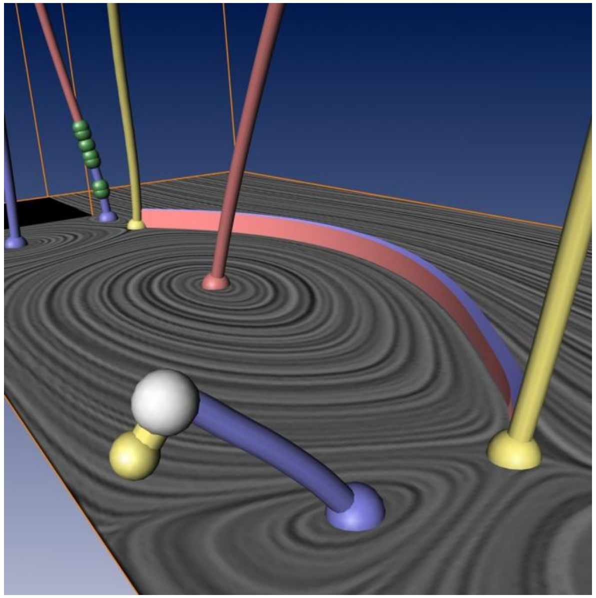


$\mathbf{p}(\mathbf{x}, t)$

Space-time Domain Approaches

- Classical theory not applicable
  - **s**(**x**,0): no isolated critical points in general
  - **p**(**x**,1): no critical points at all
  - critical structures do not coincide
  - different types of structures





Example topology network [TWHS05]

Space-time Domain Approaches

- Approach:
  - Feature Flow Field (FFF) [TS03]
    - support field in same dimension
    - points into direction of feature
  - Local definition:

$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$

$$\mathbf{f}(x,y,t) = grad(u) \times grad(v) = \begin{pmatrix} det\ (v_y, v_t) \\ det\ (v_t, v_x) \\ det\ (v_x, v_y) \end{pmatrix}$$

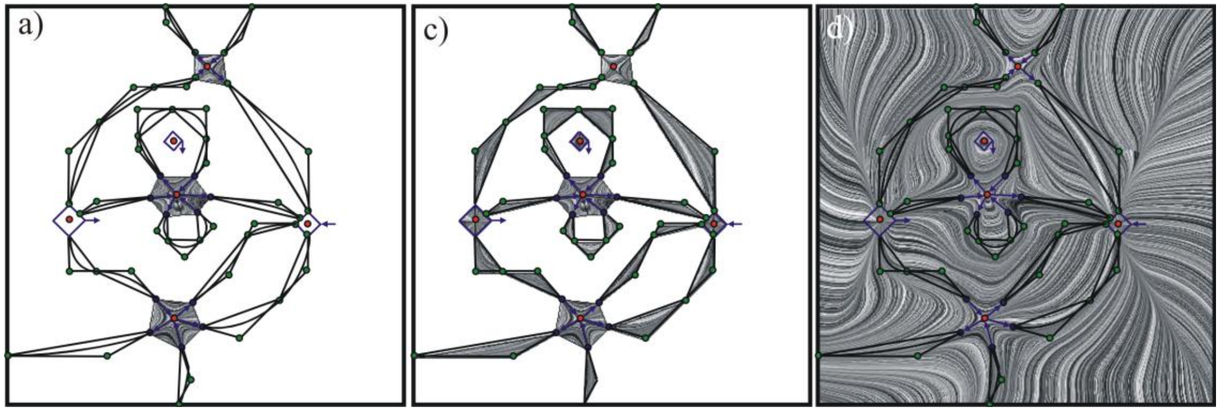
Space-time Domain Approaches

- Applications of FFF:
  - Tracking of features [TS03, TWHS04. TWHS05]
    - feature evolvement by Integration
    - critical point as slice intersection
    - integrating in **f** vs. integrating in time
    - special events:
      - split
      - merge
      - vanish

→ Localize and characterize bifurcations

Space-time Domain Approaches

- Applications of FFF:
  - topological simplification [TRS03a]
  - vectorfield compression [TRS03b]



  - extraction of vortex core lines:
    - ridges / valleys of Galilean invariant quantities [SWH05]
    - as cores of swirling particle motion [TSW05]

Space-time Domain Approaches

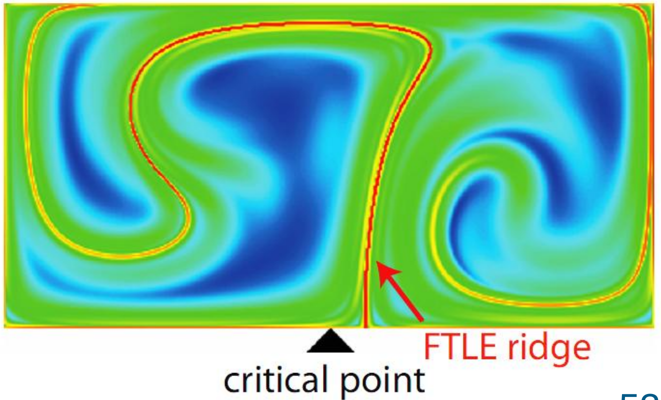
- Applications of FFF:
  - topological lines in tensor fields [ZP04,ZPP05]
    - generalization of approach
    - compact visualization and representation
  - detection of periodic behavior in LIC data [DLBB07]
    - sparse temporal sampling
    - robustness against noisy input data

On the Way Towards Topology-Based Visualization of Unsteady Flow

Local Methods

Local Methods

- Image Analysis
  - edges and ridges
  - defined pointwise, based on derivatives
- Vector field visualization
  - height ridge extraction on pressure [MK97]
  - vorticity magnitude [SKA]
  - from FTLE to find LCS [SLM05]



Local Methods

- Vector field visualization:
  - derive quantities using velocity field
  - extraction of seperation / attachement lines [KHL99]
  - vortex core lines:
    - using addtional physical quantities [BS95, MK97]
    - velocity and derivatives [LDS90, SH95]

Local Methods

- Unified local formalism: Parallel Vectors [PR99]
  - comparison to derived or additional vector data
  - can be defined for extracting lines, surfaces, ... [TSW05]
  - used to extract height ridges:
    - simplified description for any dimension
    - new class of filters [PS09b]

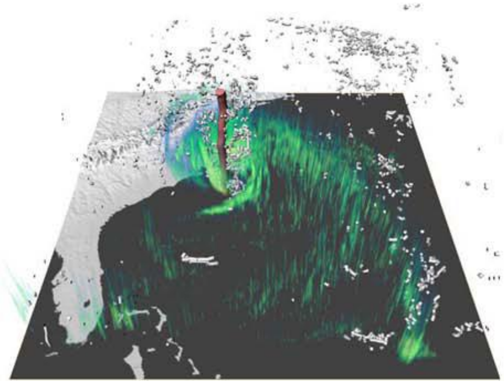
Local Methods

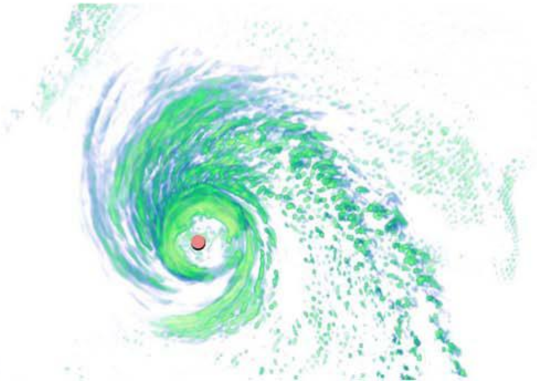
- Local methods in general
  - mostly directly applicable for time-dependent case
  - recent examples:
    - vortex core extraction for unsteady flow [WST07, FPH08]
    - reinterpretation of Sujudi & Haimes Operator [SH95]

Local Methods

- Local methods in general

  - combination with integration-based methods
  - differences to global methods [KvD93, Ebe96]
    - steady case: seperatrices only global
    - unsteady case: local definition valuable





Local Methods

- Geometric approaches
  - alternative methods for vortex detection [SP99]
    - clusters of oscilating circle centers
    - streamlines analysis
      - winding-angle
      - distance of start / end point
  - further extension to characterize 2D vortices [PKPH09]
    - detection and clustering of loop-intersections
    - parameter-free and independent of loop-geometry

On the Way towards Topology-Based Visualization of Unsteady Flow

Statistical and Multi-Field Methods

Armin Pobitzer

University of Bergen

Statistical and Multi-Field Methods

Exloring flow = consideration of

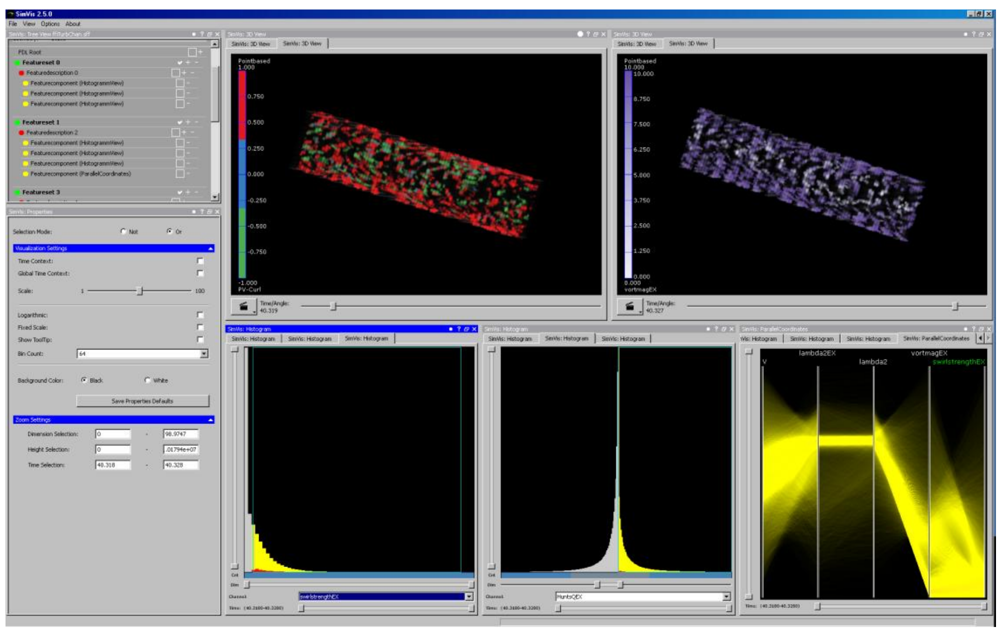
- Multiple features
- Ambiguous definitions
- Additional measures

Tools:

- Interactive Visual Analysis
- Fuzzy Feature Detectors

Interactive Visual Analysis

- Balance between automatic analysis and human percepion
- Aims to detect relations between several variabls
- Multiple views, linked views, interactive selection

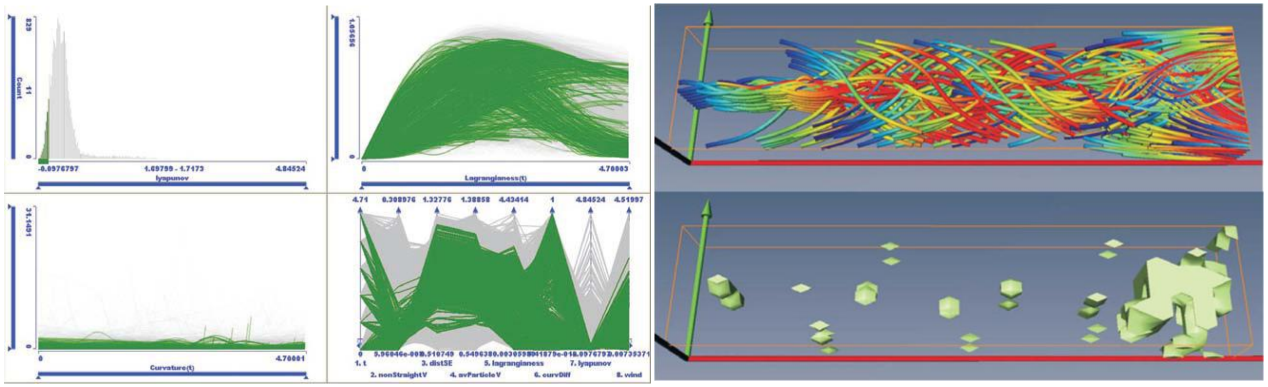


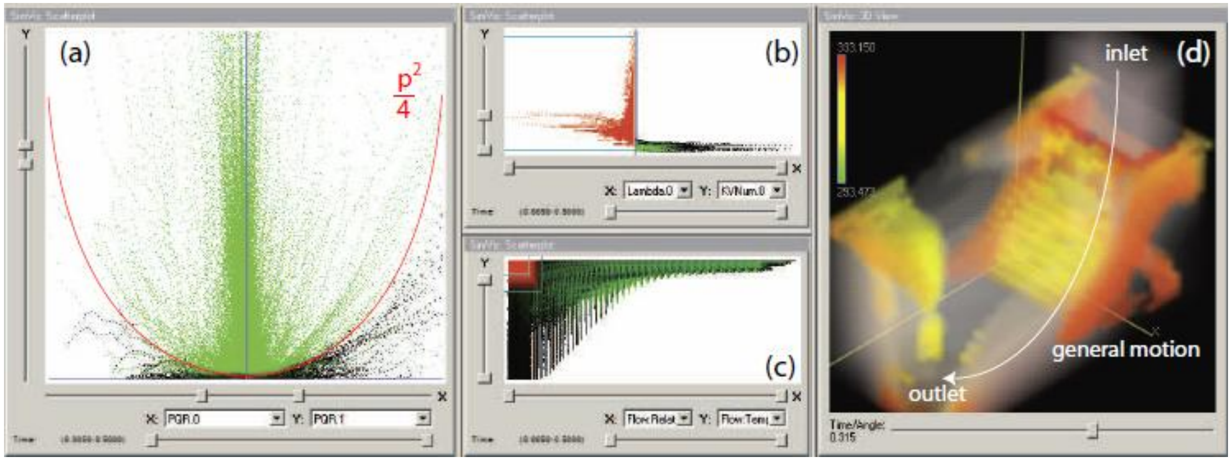
[Dol07]

Interactive Visual Analysis

- Feature detectors and other flow measures as variables [BMDH07, STH*09]

[STH*09]

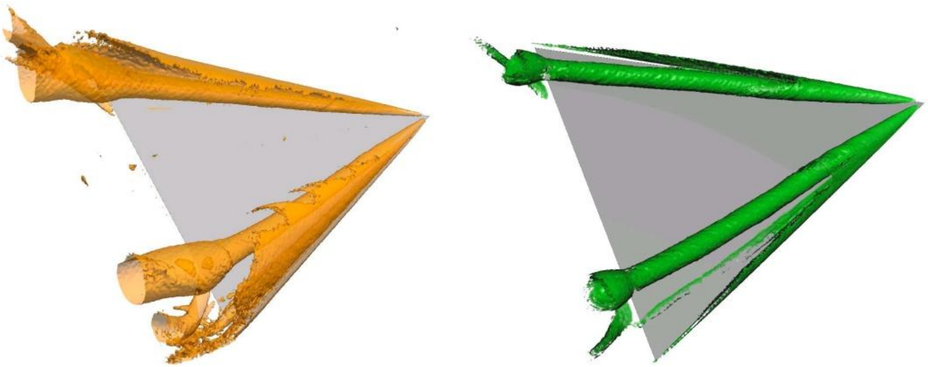




[BMDH07]

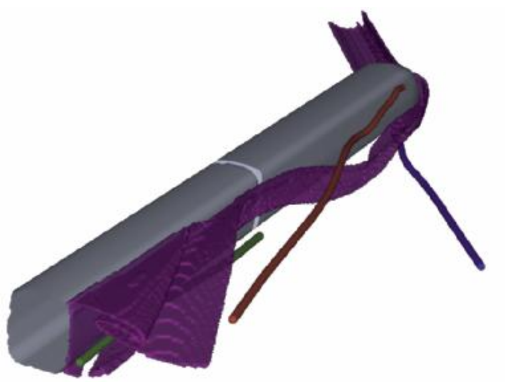
Fuzzy Feature Detector

- Automatic feature detection using statistical measure [JWSK07, JBTS08]



[JBTS08]

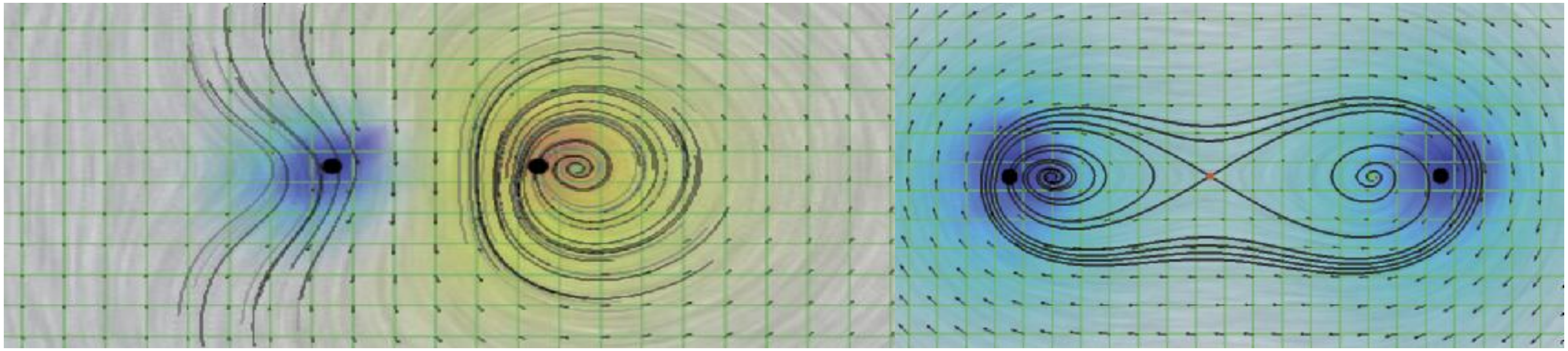
- Boolean Operaters on set of trajecories [SS07,SGSM08]



[SS07]

Fuzzy Feature Detector

- Pattern matching for feature quantification [EWGS07]



Summary

- Lagrangian methods:
  + direct physical interpretation
  - can not detect all flow structures
- Space-time domain approaches:
  + close to classical VFT
  - no unified topology description
- Local methods:
  + relatively well established
  -
- Interactive visual analysis:
  + combination of different flow aspects
  - no automatic segmentation

Conclusion

There are unsolved problems...

- No solution comparable to VFT available
- Present aproaches solve problem only partially

... but there is hope as well

- Present approaches seem to overlap
- Combination of different apraches and methods may meet the interst of the user domain better

TopoInVis


TopoInVis 2011
Fourth Workshop on Topology-Based Methods in Data Analysis and Visualization
April 4-6, 2011
Zurich, Switzerland
www.topoinvis.org
Workshop co-chairs: Ronny Peikert, Helwig Hauser, Hamish Carr
Local organization: Raphael Fuchs
Abstract submission: October 1, 2010
Paper submission: October 15, 2010
SEVENTH FRAMEWORK PROGRAMME
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Thanks for your attention!

Questions?

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